General Equilibrium Theory

- Partial equilibrium model – all prices other than the price of the good being studied are assumed to remain fixed.
- General equilibrium model – all prices are variable and equilibrium requires that all markets clear (all of the interactions between markets are taken into account)

Pure exchange model 2 consumers & 2 goods

- Pure exchange model: the special case of the GE model where all of the economic agents are consumers and nothing is neither appears nor disappears in the exchange model
- Net demander (supplier): consumer wants to consume more (less) than his endowment of that commodity

Assumptions

- the only kind of economic agent is the consumer
- no production is possible
- economic activity consists of trading and consumption
- 2 consumers ($i=1,2$) are described completely by their preferences or utility function (u_i) and 2 commodities ($k=1,2$) that they possess, i.e. initial endowment ($\omega_{ki}\geq 0$)
- consumer's preferences are continuous, strictly convex, and strongly monotone
- they trade the goods among themselves according to certain rules (price-takers)
- there is a market for each good, in which the price of that good is determined
- the goal: to make themselves better off (each consumer attempts to choose the most preferred bundle that he can afford)

Notation

- x_{ki} – consumer *i*'s consumption of commodity *k*
- $p \geq 0$ – price vector
- $x_i=(x_{1i},x_{2i})$ – consumer *i*'s consumption vector (final allocation or gross demand)
- $\omega_i=(\omega_{1i}, \omega_{2i})$ – consumer *i*'s endowment vector (initial allocation)
- $\omega_k =(\omega_{k1}+\omega_{k2})>0$ – total endowment of good *k* in the economy
- $z_i=(x_i-\omega_i)$ – consumer *i*'s excess demand
- **An allocation** in this economy is an assignment of a nonnegative consumption vector to each consumer: $x=(x_1,x_2)=((x_{11},x_{21}),(x_{12},x_{22}))$

Edgeworth box (gr. 1, 2)

- All of the information contained in a 2-person x 2-good pure exchange economy can be represented in a convenient graphical form as the Edgewoth box. Any point in the box represents a nonwasteful division of the economy's total endowment between two consumers.
- An allocation is feasible for the economy if $x_{k1}+x_{k2}\leq \bar{\omega}_{\boldsymbol{k}}$
- The fact that it is nonwasteful means that $(x_{12},x_{22})=(\bar{\omega}_1-x_{11}, \bar{\omega}_2-x_{21})$, excess demand is zero.

Offer curves (gr. 3, 4)

- How goods are allocated among the economic agents? For each *i:*

$$\max_{x_i} u_i(x_i)$$

- such that $px_i = p\omega_i$
- Offer curve – for a given endowment, it is the set of demanded bundles at every price vector

Solution

- Wealth level is determined by the values of prices:

$$\boldsymbol{W_i} = \boldsymbol{p}\omega_{\boldsymbol{i}} = \boldsymbol{p}_1\omega_{1\boldsymbol{i}} + \boldsymbol{p}_2\omega_{2\boldsymbol{i}}$$

for any vector of market prices $p=(p_1,p_2)$

- Budget set is a function of prices: $B_i(p)=\{px_i \leq p\omega_i\}$. The budget line is a line that goes through the endowment point ω with slope –(p_1/p_2). Only allocations on the budget line are affordable to both consumers simultaneously at prices (p_1 ,p_2).
- Each consumer takes these prices as given and chooses the most preferred bundle from his consumption set

Competitive (Walrasian) equilibrium

- <u>Competitive (Walrasian) equilibrium</u> for an Edgeworth box economy is a pair *(p*, x*)* such that

$$\sum_i \boldsymbol{x}_i(\boldsymbol{p}^*, \boldsymbol{p}^*\omega_i) \leq \sum_i \omega_i$$

- p* is a competitive equilibrium, if there is no good for which there is a positive excess demand ($\sum_i z_i(p) \leq 0$)
- if one consumer whishes to be a net demander of some good, the other must be a net supplier of this good in exactly the same amount
- demand should equal supply, if all goods are desirable

More on Walrasian equilibrium

- At an equilibrium in the Edgeworth box the offer curves of the two agents intersect. At such an intersection the demanded bundles of each agent are compatible with the available supplies. (**gr. 5**)
- If $p^* = (p_1^*, p_2^*)$ is a competitive equilibrium price vector, then so is $\alpha p^* = (\alpha p_1^*, \alpha p_2^*)$ for any $\alpha > 0$.
- only relative prices p_1^*/ p_2^* are determined in an equilibrium
- to determine equilibrium prices we need only to determine prices at which one of the markets clears; the other market will necessarily clear at these prices
- It may happen that a pure exchange economy does not have any Walrasian equilibria if one of the consumers preferences are:
  - not strongly monotone and the endowment lies on the boundary of the Edgeworth box (**gr.6a**)
  - nonconvex (**gr. 6b**)

Pareto optimality (7 a,b,c)

- <u>Pareto optimal (efficient) allocation</u> – an allocation where there is no alternative feasible outcome at which every individual in the economy is at least as well off and some individual is strictly better off (no matter of a market type)
- At any competitive allocation, there is no alternative feasible allocation that can benefit one consumer without hurting the other
- Hence, any competitive equilibrium allocation is Pareto optimal, it lies in the **contract curve** portion of the Pareto set
- <u>First fundamental theorem of welfare economics in the Edgeworth box:</u> Competitive allocations yield Pareto optimal allocations (**gr. 8**)

Second fundamental theorem

- Second fundamental theorem of welfare economics in the Edgeworth box says that under convexity assumptions (not required for the first welfare theorem), any desired Pareto optimal allocation can be achieved by appropriately redistributing wealth in a lump-sum fashion and then letting the market work (i.e. any Pareto optimal allocation is supportable as an equilibrium with transfers). It means that some Pareto optimal allocations may not be a competitive equilibria, unless we transfer wealth.
- An allocation x^* in the Edgeworth box is an equilibrium with transfers if there is a price system p^* and wealth transfers T_1 and T_2 satisfying $T_1+T_2=0$ (i.e. the planner runs a balanced budget, only redistributing wealth between the consumers), such that for each consumer i we have (**gr. 9**): $u_i(x^*) > u_i(x')$ for all x' such that $p^* x'_i \leq p^* \omega_i + T_i$